SHELL-WORLD

A NEW MEGASTRUCTURE “FOR SALE”

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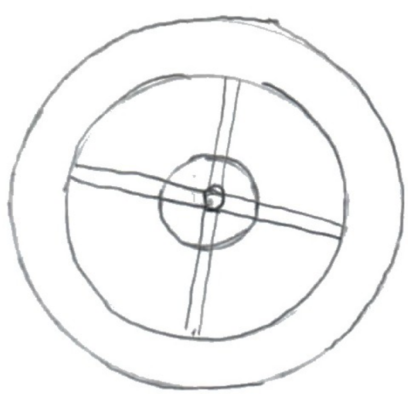
This is a write-up of an interesting idea I had back in August 2020. It’s for a particular type of sci-fi megastructure which should be stable (at least to back-of-the-envelope calculations) and would make an interesting setting/location/super-architecture-style for any authors who are interested. Not practical to build anytime soon, but interesting.

I’m not sure if this is original or not (I haven’t looked very hard.) If it is, I’m staking my claim! Publish or perish!

Another Update: Apparently this isn’t very clear: I have two main sections – the first is about how large unsupported spin-gravity structures can get. The second is about the shell-world, whose inhabitants live in almost zero gravity. I figure by the time we’re smelting the entire crust of the moon to build artificial planetoids, we can handle a little genetic engineering to deal with living in zero gravity.

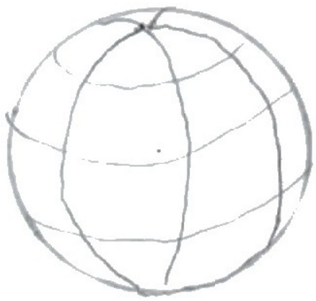
Spin Gravity Digression

I came up with the idea while thinking about ring-stations and other spin-gravity structures. Spin gravity is a very practical, easily attained way of generating “artificial gravity” in which to live/work/escape weightlessness.



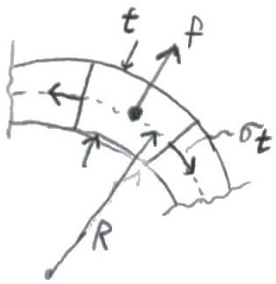
How big can you make these things, given construction materials of various types?

Without any forces, you can make structures as big as you want.



Hollow planet made out of tinfoil, barely pressurized to balance gravity acting on the foil.
(Genesis of the idea → we'll come back to this)

However, in a spin-gravity structure, you **do** have a source of structural stress: The centrifugal force acting on the structure.



$f = \rho t g_s = \rho t \omega^2 r$ Force per unit area $[N/m^2]$

$g_s = \omega^2 R$ Spin gravity acceleration $[m/sec^2]$

σ_t – tensile stress in the structure $[N/m^2]$

ω – angular rotation rate of the structure [rad/sec]

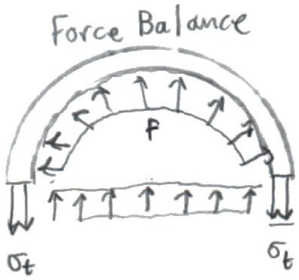
R – radius of structure centerline [m]

t – thickness of structure outer wall [m]

Digression to this digression: Yes, centrifugal **force**. If you look at the frame-forces you have in a non-inertial rigid reference frame, you get not just centrifugal force, but Coriolis and Euler forces, which are also important. Coriolis forces are important because they cause sideways pulls when you move relative to the spinning frame: Possibly disorienting to astronauts standing up quickly, or throwing a ball. Generally speaking, the larger your ring, the slower it can rotate to generate a given amount of spin gravity, and the less Coriolis distortion you have to deal with.

Grade-school pedants will object to centrifugal force as a valid force, insisting that all thinking be done in the straightjacket of inertial frames: But by that standard, gravity is also a fictitious force due to calculating in a non-inertial frame: Pretending space-time is Euclidean in situations where it isn't. The mass-proportionality of each of these forces is what prompted Einstein to try tying these all together.

Because this is a back-of-the-envelope calculation, I want to make my life as easy as possible. I'm considering only rings, thin relative to their major radii, without giant tubes filled with gas and water and orbital offices, etc. This simple scenario should provide an upper bound on how large a ring-structure can get.



$$2Rf = 2\sigma_t t$$ Force per unit length along axis [N/m]

$$\sigma_t = \frac{fR}{t} = \frac{t\rho\omega^2 R\cdot R}{t} = \rho\omega^2R^2$$ tensile stress in structure [N/m²]

Thickness divides out!

It turns out that in this simplified scenario, the structural thickness divides out. It doesn't matter how thick your ring becomes. More thickness provides more area to divide the stress over, but also proportionally more weight under spin-gravity. The rings care only about how dense the material is, how large the ring is, and how fast it is rotating. This points to a maximum size limit for a given material: The figure of merit ends up being tensile strength over density, called specific tensile strength.

$$R_{max} = \frac{\sigma_{t,Y}}{\rho\, g_s SF}$$ SF - a safety factor to the stress on the ring

$$g_s = \omega^2 R_{max}$$

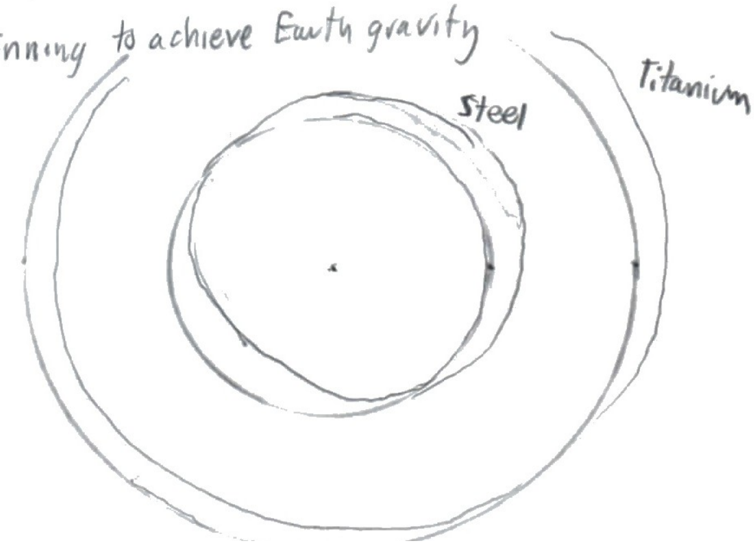
Some materials and yield stresses [engineeringtoolbox.com]

Material	ρ [kg/m³]	E [GPa]	$\sigma_{tensile}$ [MPa]	σ/ρ [MPa/(kg/m³)]
AISI 1045 Steel	7700	205	585	0.076
Aluminum 6061-T6	2700	69	270	0.100
Aluminum 2045-T4	2700	73	450 ?	0.166
Titanium Alloy	4500	105	730 (yield)	0.162

Ignoring the safety factor: SF = 1

$g = 9.8\ m/sec^2$ - ring spinning to achieve Earth gravity

Material	Rmax [m]
Steel	7752 m
Al 6061-T6	10,204 m
Al 2045-T4	17,006 m
Titanium	16,533 m

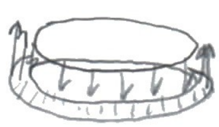


In science fiction, there are many extremely large spinning habitats: Larry Niven's ringworld postulates the existence of some nearly indestructible material to hold together his planetary-orbit-radius massive ring. With more realistic materials you have to spin slower for less gravity if you want to build a bigger unsupported ring structure.

Pressure Supported Shell World

So back to the idea of a balloon megastructure: If you have pressure inside a sphere, you (usually) need to balance the pressure with tension in the wall.

Force balance: pressure vs. tension



$$p \cdot \pi R^2 = \sigma_t \cdot 2\pi R t$$

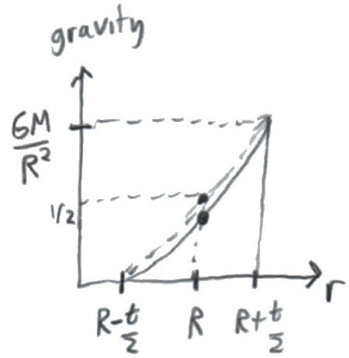
$$\sigma_t = \frac{pR}{2t} \quad \left[\frac{N}{m^2}\right]$$ tensile stress in the shell

R - shell radius [m]

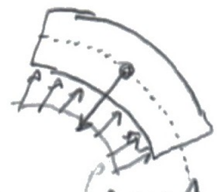
t - shell thickness [m]

p - internal pressure [Pa] or [N/m^2]

This becomes a problem if the shell becomes very large. However, gravity can help us here. In fact, if there is enough gravity, it can cancel the tension in the structure (to back-of-the-envelope-order) entirely. (Or almost entirely, you may want some residual stress left over for buckling stability.)



Force balance: Pressure vs avg gravitational force (roughly $\frac{1}{2}$ at centerline)



$G = 6.67E\text{-}11 \left[\frac{N\text{-}m^2}{kg^2}\right]$ Newtons gravity constant

$f_p = pA$ Force of pressure on a given area

$f_g = \rho A t \cdot g$ Force of gravity on a given area

$g \approx \frac{GM}{R^2} \cdot \frac{1}{2}$ Force of gravity on shell centerline

$M = 4\pi R^2 t \cdot \rho$

$g \approx 4\pi G \rho t \cdot \frac{1}{2}$ This depends on shell <u>thickness</u>, <u>not</u> on shell radius!

The gravitational force acting on a bit of matter on the surface of a solid sphere is the same as the gravitational force that would result from any spherically-symmetric distribution of that matter. The mass could all be concentrated in the center in a point, or it could be concentrated in the surface of the shell. As the shell radius becomes large relative to the thickness, the variation of the gravitational force (starting at zero on the interior) becomes nearly linear across the cross-section of the shell.

$pA = \rho t A \cdot g$

$p = (4\pi G \cdot \frac{1}{2}) \cdot \rho^2 t^2$

$t = \sqrt{\frac{p}{(4\pi G \cdot \frac{1}{2}) \cdot \rho^2}}$ thickness to gravitationally balance internal pressure

(Unit check)

$\sqrt{\frac{N/m^2}{\frac{N\text{-}m^2}{kg^2} \cdot \frac{kg^2}{m^6}}} = \sqrt{m^2} = m$ ✓

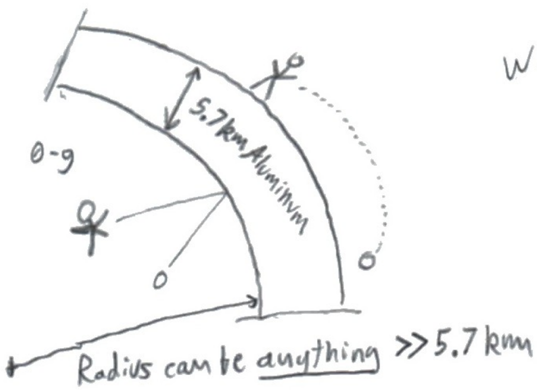
Example:

$\rho_{Al} = 2700 \frac{kg}{m^3}$ Aluminum is a common metal: 8% of Earths crust by mass

$G = 6.67E\text{-}11 \frac{N\text{-}m^2}{kg^2}$

$P_{atm} = 101325\ N/m^2$

$$t = \sqrt{\frac{101325\ Pa}{(2\pi \cdot 6.67E\text{-}11 \frac{N\text{-}m^2}{kg^2} \cdot (2700\ kg/m^3)^2}} = \boxed{5758m}$$



What is surface gravity?

$g = 4\pi t \rho G$

$g = 0.01302\ m/sec^2$

$= 0.0013 g_e$ – not much!

Update: The radius can be as large as 100,000km, after which the gravitation of the enclosed mass of air starts to change the picture. (See two sections down.) Between 100s km and gas-giant-scale this concept appears to work.

Thinking A Bit More About The Shell-World

Let's suppose the aluminum shell-world were something like 250km in radius: That's about 1.20E19 kg of aluminum required to build the thing. Quite a bit of material! However, the mass of Earth's moon is 7.34E22 kg, so this would be about 0.016% of the mass. If we wanted to mine the moon to build one of these, it might only account for 2% of the aluminum we could extract from the moon.

A 250km radius shell-world would contain 65 million cubic kilometers of space. Am I justified in assuming near-zero-gravity inside a hollow shell-world? The gravity due to the mass of air at STP inside would be 8.5E-5 m/sec2. This is negligible, and because of this, any nonuniformities of the pressure in the interior of the shell would not be due to gravity. There would be no variation in pressure with "altitude".

The obvious reason to build one of these things is **extreme** amounts of living room in a compact structure. The shell-world would likely be filled with a lattice of structures to help fill out and define all that near-zero-gravity interior. Let's imagine a bunch of 100m diameter skyscrapers forming a cubic lattice, 10km on a side. Each of these cubes would be about 0.09% tower by volume. If the towers were solid aluminum (unlikely!) they would provide enough mass to give 0.0001 m/sec^2 additional gravity at the surface. As long as the interior isn't built up too densely, the interior won't perturb the gravitational distribution of the object as a whole. There would be an extremely gentle pressure causing things to drift slowly towards the center, but this effect almost vanishes as the center approaches, and terminal speeds would be very slow.

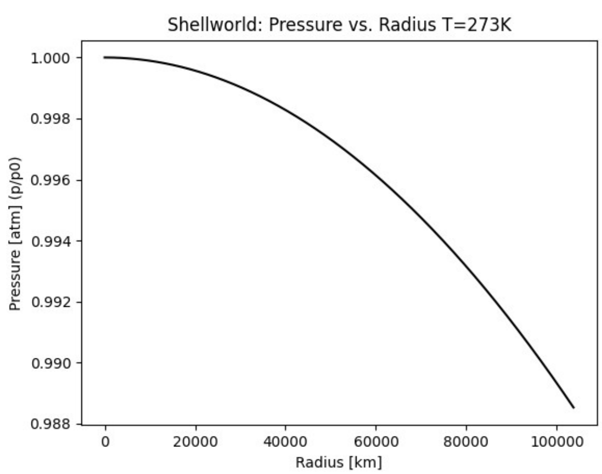
On Earth, we have a surface area of 511 million km2 (including oceans), but most of our lives are lived within one km or less of the surface. This compact 500km diameter sphere could easily contain a nontrivial fraction of the living space in the entire Earth.

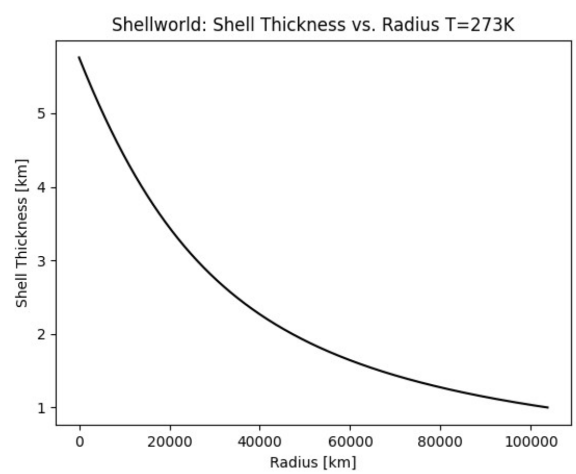
A Correction:

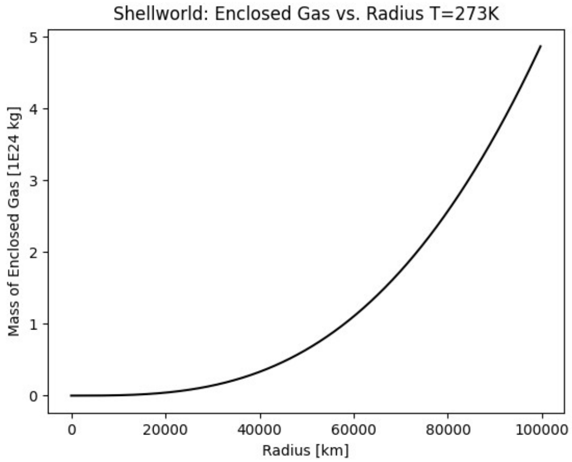
It was pointed out to me by a friend, and one other acquaintance on a forum, that this concept in fact cannot scale to arbitrarily large (Dyson-sphere-type) sizes. Somewhere between the 100s km scale that I was thinking of, and gas giant scale, the gravitation of the contained gas causes the pressure to decrease at the edge, and the gravity reduces the size of the shell that can be supported by that pressure.

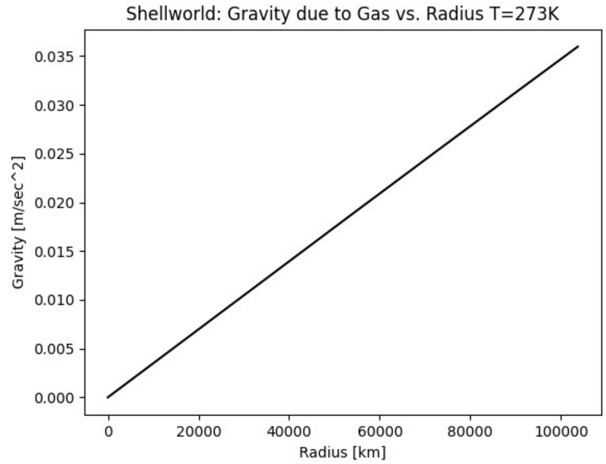
I've written a python script to integrate the relevant equations, and have found that this occurs at about a scale of 100,000 km, for T=273K interior gas temperature, for a central pressure of 101325 Pa.

Here is roughly how all of these variables behave with radius:









Python Script

```
#!/usr/bin/python

import os,sys,math
import numpy as np
import matplotlib.pyplot as plt

pi = np.pi

G = 6.67E-11 #N-m^2/kg^2
Na = 6.022E26 # #/kg
kb = 1.3806E-23 # J/K (J/#-K)
Ru = kb*Na #8314 J/kmol-K

Mair = 28
Rg = Ru/Mair
p0 = 101325
T = 273
rhoal = 2700

def thfunc(g0,rho,p):
    G = 6.67E-11 #N-m^2/kg^2

    a = 2*pi*G*rho**2
    b = g0*rho
    c = -p

    t = (-b + np.sqrt(b**2-4*a*c))/(2*a)

    return t

Rmax = 200000*1000
N0 = 1000000
dr = Rmax/N0
rhol = np.zeros((N0))
gl = np.zeros((N0))
ml = np.zeros((N0))
pl = np.zeros((N0))
tl = np.zeros((N0))
rl = np.zeros((N0))

rr = 0
pl[0] = p0
rhol[0] = p0/Rg/T
ml[0] = 0.0
gl[0] = 0.0
tl[0] = thfunc(0.0,rhoal,pl[0])
rl[0] = 0.0
```

```
for I in range(1,N0):
    rr = rr + dr
    dV = 4*pi*rr**2*dr

    p = pl[I-1]
    rho = p/Rg/T
    ml[I] = ml[I-1] + rho*dV
    gl[I] = G*ml[I]/rr**2
    pl[I] = pl[I-1] - rho*gl[I]
    rhol[I] = rho
    tl[I] = thfunc(gl[I],rhoal,pl[I])
    rl[I] = rr

    if(tl[I]<1000.0 or pl[I]<0.0):
        break

N = I
ml.resize((N))
gl.resize((N))
pl.resize((N))
gl.resize((N))
rhol.resize((N))
tl.resize((N))
rl.resize((N))

print(pl[0])
print(tl[0])
print(pl[N-1])
print(tl[N-1])
print(gl[0])
print(gl[N-1])

plt.figure()
plt.plot(rl/1000,pl/p0,'k-')
plt.title('Shellworld: Pressure vs. Radius T=273K')
plt.xlabel('Radius [km]')
plt.ylabel('Pressure [atm] (p/p0)')

plt.figure()
plt.plot(rl/1000,tl/1000,'k-')
plt.title('Shellworld: Shell Thickness vs. Radius T=273K')
plt.xlabel('Radius [km]')
plt.ylabel('Shell Thickness [km]')

plt.figure()
plt.plot(rl/1000,gl,'k-')
plt.title('Shellworld: Gravity due to Gas vs. Radius T=273K')
plt.xlabel('Radius [km]')
plt.ylabel('Gravity [m/sec^2]')

plt.figure()
plt.plot(rl/1000,ml/1E24,'k-')
plt.title('Shellworld: Enclosed Gas vs. Radius T=273K')
plt.xlabel('Radius [km]')
plt.ylabel('Mass of Enclosed Gas [1E24 kg]')

plt.show()
```